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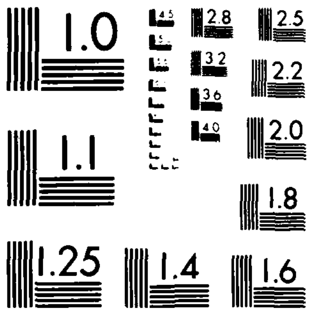

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Nonlocal Continuum Theory for Dislocation and Fracture

A.C. Eringen
PRINCETON UNIVERSITY

Technical Report No. 61
Civil Engng Res. Rep. No. 84-SM-2

PRINCETON UNIVERSITY
Department of Civil Engineering

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ABSTRACT (cont)

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Princeton University

NONLOCAL CONTINUUM THEORY FOR DISLOCATION AND FRACTURE

A.C. Eringen

ABSTRACT - By means of linear theory of nonlocal elasticity, solutions are given for some Voltera dislocations situated in an elastic solid. The stress fields are determined for screw and edge dislocations. The stresses and elastic energy are devoid of usual singularities predicted by the classical (local) elasticity. A theory is developed for continuous distributions of dislocations on the basis of nonlocal incompatible elasticity. Stress fields are given for volume, surface and line distributions of dislocations. Celebrated Peach-Koehler formula is modified to include nonlocal Green's functions. The stress fields for three- and two-dimensional cases and for the anti-plane strain are determined for line distributions. Calculations are carried out for the uniform distributions of edge and screw dislocations along a straight line segment. By means of the maximum stress hypothesis, a fracture criteria is introduced. Calculated theoretical strengths are in good agreement with those based on the atomic models. Reduction of material strength with the presence of dislocation line and the maximum number of dislocations are given.

IT IS A WELL-KNOWN FACT that stress fields due to Volterra dislocations, contain singularities at the center of the dislocation so that in a small region around the center (the core region) classical elasticity fails to apply. The radius of this region is estimated, usually by means of atomic models. Because of these stress and energy singularities, several other methods have been devised for the prediction of fracture. Clearly such singularities are non-physical and a proper model should eliminate them.

Phonon dispersion experiments have shown that the phase velocity of plane waves in single crystals depends on the wave length so that dispersion is the rule rather than the exception. Yet classical elasticity predicts constant phase velocities for plane waves, independent of frequency and wave length.

There are many other physical phenomena in the microscopic scale, that cannot be predicted by means of classical elasticity (linear or nonlinear). Among these, we mention the state of stress at a sharp crack tip, surface tension, atomic inclusions, defects, granular, porous and composite solids.

In several previous papers [cf., 1-4], we have shown that the stress fields, due to dislocations and cracks, predicted by the nonlocal elasticity contains no singularities. In fact, they vanish at the center of dislocations and at the crack tip. Moreover, the maximum stress occurs at a short distance away from these points. By equating the maximum stress to the cohesive stress that holds atomic bonds together, a physically realistic fracture criteria was established[3,5]. In the classical limit, the celebrated Griffith criterion is obtained with the dividend that the Griffith constant is determined without any additional assumption on the surface energy which could not be measured to within any reasonable accuracy. Estimated errors in such measurements is known to be not less than several hundred percent.

The dispersion curves predicted by the nonlocal theory are nearly in coincident with those based on the atomic lattice dynamics and observations on phonon dispersions [cf. 4-6].

There exist ample evidence that recently developed theory of nonlocal elasticity[6-9] is a proper mathematical model which can eliminate the foregoing difficulties, extending the domain of applicability of continuum mechanics to physical phenomenon with internal characteristic lengths at the molecular, atomic or microstructural scales.

Based on these observations, we expect that a nonlocal theory may bear fruit in dealing self-stress and energies of dislocation loops for both the discrete and continuous distributions of dislocations. Moreover, the onset of fracture and the theoretical strength of solids may be estimated by means of the continuum theory which permit extensions to amorphous solids and composites. The raison d'être of the present paper stems from these observations.

In section 2, I present a summary of the linear theory of nonlocal isotropic elastic solids. In section 3, the stress field and energy due to a screw dislocation are calculated. Both turned out to be devoid of singularities. The stress field due to an edge dislocation is treated in section 4. In section 5, I develop a theory for the continuous distribution of dislocations. Field equations are obtained for stress functions for two and three-dimensional state of strain.

Green's functions for the infinite solids are obtained in section 6 leading to a generalized Peach-Koehler formula for the stress field due to line distribution of dislocations. Results are gratifying in that they contain no singularities so that calculations can be carried out with uniformity, for surface and line distributions of dislocations throughout core regions. In fact, stress fields due to a uniform distribution of screws along a line segment verifies our predictions. The theoretical strength of a single crystal predicted by the nonlocal theory is in agreement with that known in atomic theory. Also given are the shear strengths reduction due to line distributions and the maximum allowable number of screws within a straight line segment of length 2ℓ.

2. BASIC EQUATIONS

From the atomic theory of lattice dynamics and experimental observations on phonon dispersions, it is well-known that the stress at a material point $\underset{\sim}{x}$ in a body depends not only on strains at $\underset{\sim}{x}$ but also on strains at <u>all</u> other points $\underset{\sim}{x}'$ of the body. In linear theory of nonlocal elasticity, this is expressed by an integral constitutive equation of the form [cf. 2,4].

$$(2.1)\qquad t_{k\ell} = \int_V c_{k\ell mn}(\underset{\sim}{x}'-\underset{\sim}{x})\, e_{mn}(\underset{\sim}{x}')\, dv(\underset{\sim}{x}')$$

where $c_{k\ell mn}$ are the material property functions which depend on the vector $\underset{\sim}{x}'-\underset{\sim}{x}$ and $e_{k\ell}$ is the linear strain measure defined by

$$(2.2)\qquad e_{k\ell}(\underset{\sim}{x}') = \frac{1}{2}\left[\frac{\partial u_k(\underset{\sim}{x}')}{\partial x'_\ell} + \frac{\partial u_\ell(\underset{\sim}{x}')}{\partial x'_k}\right]$$

where $u_k(\underset{\sim}{x},t)$ is the displacement vector. For homogeneous and isotropic solids, (2.1) may be simplified to

$$(2.3)\qquad t_{k\ell}(\underset{\sim}{x}) = \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\, \sigma_{k\ell}(\underset{\sim}{x}')\, dv(\underset{\sim}{x}')$$

where $\sigma_{k\ell}$ is the classical (local) stress tensor given by Hooke's law:

$$(2.4)\qquad \sigma_{k\ell}(\underset{\sim}{x}') = \lambda\, e_{rr}(\underset{\sim}{x}')\,\delta_{k\ell} + 2\mu\, e_{k\ell}(\underset{\sim}{x}')$$

in which λ and μ are the usual Lamé constants. In Eq. (2.3), the kernel $\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ is an influence function (*attenuation function*) which brings influences of strains at various points $\underset{\sim}{x}'$ to $\underset{\sim}{x}$, in different proportions. According to the *attenuating neighborhood hypothesis*[10], it assumes the maximum value at $\underset{\sim}{x}'=\underset{\sim}{x}$, sharply decreasing with the distance from $\underset{\sim}{x}$. From Eq. (2.3), it is clear that α depends on a length scale ε. This is an internal characteristic length which may be selected to be proportional to the lattice parameter a for single crystals, i.e.

$$(2.5)\qquad \varepsilon = e_0 a\ ,$$

average granular distance for amorphous bodies and the average distance for fiber composites, etc. In Eq. (2.5), e_0 is a non-dimensional constant which can be determined by one experiment.

When $\varepsilon \to 0$, Eq. (2.3) must revert to (2.4). This implies that α is a Dirac delta sequence. Thus, in this limit nonlocal theory reverts to the classical elasticity theory. By discretizing Eq. (2.3), it can be shown that equations of nonlocal elasticity also reduce to those of atomic lattice dynamics[11].

In several previous papers[4,12,13], I gave special representations for $\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ which lead to excellent predictions in accord with the atomic lattice dynamics. For example, for the two-dimensional case, an appropriate kernel is

$$(2.6)\qquad \alpha(|\underset{\sim}{x}|) = (2\pi\varepsilon^2)^{-1} K_0(\sqrt{\underset{\sim}{x}\cdot\underset{\sim}{x}}/\varepsilon)$$

where K_0 is the modified Bessel's function. For two-dimensional lattices, Eq. (2.6) provides an excellent match between acoustical dispersion curves, based on the nonlocal elasticity and those based on Born-Kármán theory of atomic lattice dynamics. In the entire Brillouin zone, the error is less than 6%. It is also interesting to note that for the infinite medium α is the Green's function of a linear differential operator. In the case of Eq. (2.6), this means that α satisfies

$$(2.7)\qquad (1-\varepsilon^2\nabla^2)\alpha = \delta(|\underset{\sim}{x}'-\underset{\sim}{x}|)$$

vanishing at infinity.

Under the mild assumptions of vanishing nonlocal effects for the body forces and couples, the momentum balance laws of nonlocal elasticity reduces to Cauchy's laws

$$(2.8)\qquad t_{k\ell,k} + \rho(f_\ell - \ddot{u}_\ell) = 0$$

$$(2.9)\qquad t_{k\ell} = t_{\ell k}$$

where ρ is the mass density and f_ℓ is the body force density.

As usual, repeated indices denote summation over the range of indices. We also abreviate partial differentiation with respect to x_k by an index following a comma, and use a superposed dot to indicate the time derivative, e.g.

$$t_{k\ell,k} = \frac{\partial t_{k\ell}}{\partial x_k}\ ,\qquad \dot{u}_\ell = \frac{\partial u_\ell}{\partial t}$$

The field equations of nonlocal elasticity are obtained by combining (2.1), (2.2) and (2.8).

$$(2.10)\quad -\int_{\partial V} c_{k\ell mn}(\mathbf{x}'-\mathbf{x})\, e_{mn}(\mathbf{x}')\, da'_k$$
$$+\int_{V} c_{k\ell mn}(\mathbf{x}'-\mathbf{x})\, u_{m,nk}(\mathbf{x}')\, dv'$$
$$+\ \rho(f_\ell - \ddot{u}_\ell) = 0$$

where a superposed prime denotes dependence on $\mathbf{x}'$, e.g. $u'_m = u_m(\mathbf{x}')$, $dv' = dv(\mathbf{x}')$. In deriving (2.10), we used the identity

$$\frac{\partial}{\partial x_k}(c_{k\ell mn}e'_{mn}) = -\frac{\partial c_{k\ell mn}}{\partial x'_k}e'_{mn}$$
$$= -\frac{\partial}{\partial x'_k}(c_{k\ell mn}e'_{mn}) + c_{k\ell mn}\frac{\partial e'_{mn}}{\partial x'_k}$$

and the Green-Gauss theorem to convert the first term to the surface integral over ∂V in (2.10). In (2.10), the first integral represents the surface stresses (e.g. surface tension). Consequently, nonlocal theory accounts for the *surface physics*. This important asset of nonlocal theory is not included in classical field theories.

The integro-partial differential equations (2.10) replaces Navier's equations of classical elasticity. The displacement field $\mathbf{u}$ is to be determined by solving (2.10) under appropriate initial and boundary conditions. Boundary and initial conditions on displacement and velocity fields are identical to those of classical elasticity. Boundary conditions on tractions is based on the true *stress tensor* $t_{k\ell}$, not on $\sigma_{k\ell}$, i.e.

$$(2.11)\qquad t_{k\ell}n_k = t_{(n)\ell}$$

where $t_{(n)\ell}$ are prescribed boundary tractions. For mixed boundary-value problems, to avoid a possible overspecification, care is necessary along the boundary of the two surfaces on one of which u_ℓ and on the other $t_{(n)\ell}$ are prescribed[14].

If we assume that the nonlocal kernel α satisfies (2.7), then for the homogeneous and isotropic solids of infinite extent, integro-differential equations (2.10) can be replaced by *singularly perturbed* partial differential equations. This is achieved by noting

$$(2.12)\qquad (1-\epsilon^2\nabla^2)t_{k\ell} = \sigma_{k\ell}$$

and using (2.9) in (2.12):

$$(2.13)\qquad (\lambda+\mu)u_{k,k\ell} + \mu u_{\ell,kk}$$
$$+\ (1-\epsilon^2\nabla^2)(\rho f_\ell - \rho\ddot{u}_\ell) = 0$$

In the static case and vanishing body forces (2.13) reduce to Navier's equations

$$(2.14)\qquad (\lambda+\mu)u_{k,k\ell} + \mu u_{\ell,kk} = 0$$

However, note that the stress field is determined by solving (2.12).

3. SCREW DISLOCATION

A screw dislocation is introduced by cutting a solid along the plane $x_2 = 0$, $x_1 \geq 0$ and introducing a constant displacement discontinuity b (called Burger's vector along the x_3-direction of the rectangular coordinates. In this case, the displacement field has only single component $u_3(x_1, x_2, t)$ which is determined by solving

$$(3.1)\qquad \nabla^2 u_3 = 0$$

The stress field is obtained by solving (2.12), where σ_{31} and σ_{32} are the only non-vanishing components of the local stress tensor. These and u_3 are given by[15]:

$$(3.2)\qquad u_3 = \frac{b}{2\pi}\theta\ ,$$

$$(3.3)\qquad \sigma_{31} = -\frac{\mu b}{2\pi r}\sin\theta,\qquad \sigma_{32} = \frac{\mu b}{2\pi r}\cos\theta$$

where (r,θ,z) are the cylindrical coordinates, i.e.

$$(3.4)\qquad x_1 = r\cos\theta\ ,\qquad x_2 = r\sin\theta,\quad x_3 = z$$

To obtain the stress tensor $t_{k\ell}$, we determine the solution of (2.12) under the condition that t_{zr} must vanish on a circular cylindrical surface of radius r, $-\pi/2 < \theta < \pi/2$ as $r \to \infty$. Such a solution was given in a previous paper[7].

$$(3.5)\qquad t_{z\theta} = \frac{\mu b}{2\pi r}\left[1 - \frac{r}{\epsilon}K_1(r/\epsilon)\right],\qquad t_{zr} = 0$$

This is regular for all $0 \leq r \leq \infty$. It is valid in the core region. In fact, for $r = 0$, $t_{z\theta}$ vanishes so that the hoop stress is zero at the center of the dislocation, a result which is in accord with the physics of the problem.

Classical elasticity solution results in the case of $\epsilon = 0$, leading to a stress singularity r^{-1}. The strain energy per unit length of x_3, in a region bounded by concentric cylinders of radius r_0, and R, is given by

$$(3.6)\qquad E_L = 2\pi\int_{r_0}^{R} t_{z\theta}\, e_{z\theta}\, r\, dr$$
$$= \frac{\mu b^2}{4\pi}\left[\ln(R/r_0) + K_0(R/\epsilon) - K_0(r_0/\epsilon)\right]$$

Contrary to the classical result, this has no singularity for $r_0 = 0$. In fact,

$$(3.7) \quad \Sigma/L\Big|_{r_0=0} = \frac{\mu b^2}{8\pi}\left[\ln(R/2\varepsilon) + K_0(R/\varepsilon)\right]$$

which makes sense on physical grounds. For large R, Eq. (3.7) may be approximated by

$$(3.8) \quad \Sigma/L\Big|_{r_0=0} = \frac{\mu b^2}{8\pi}\left[\ln(R/2\varepsilon) + (\pi\varepsilon/2R)^{\frac{1}{2}} \exp(-R/\varepsilon)\right]$$

which exhibits the dependence of the energy on the size of the solid.

4. EDGE DISLOCATION

A straight edge dislocation in a solid causes plane strain with displacement vectors characterized by

$$(4.1) \quad u_1 = u_1(x_1,x_2); \quad u_2 = u_2(x_1,x_2), \quad u_3 = 0$$

where u_k are the rectangular components of the displacement vector. The displacement component u_1 undergoes a constant jump discontinuity b along the half plane $x_1 \geq 0$, $x_2 = 0$. The classical elasticity solution of this problem is well-known[15]. For convenience, we write the classical stress in the form

$$(4.2) \quad \begin{aligned} \Theta_c &= \sigma_{11} + \sigma_{22} = iBr^{-1}(e^{i\theta} - e^{-i\theta}), \\ \Phi_c &= \sigma_{22} - \sigma_{11} + 2i\,\sigma_{12} = iBr^{-1}(e^{-i\theta} + e^{-3i\theta}) \end{aligned}$$

where

$$(4.3) \quad B = \mu b/2\pi(1-\nu)$$

From Eq. (2.12), it follows that the true stress field satisfies

$$(4.4) \quad (1 - \varepsilon^2\nabla^2)\{\Theta,\Phi\} = \{\Theta_c, \Phi_c\}$$

where Θ and Φ have the forms

$$(4.5) \quad \Theta = t_{11} + t_{22}, \qquad \Phi = t_{22} - t_{11} + 2it_{12}$$

Thus, we must find the general solution of

$$(4.6) \quad (1-\varepsilon^2\nabla^2)F = (\varepsilon r)^{-1} e^{in\theta}, \qquad n = \pm 1, -3$$

The solution of (4.6) which is regular at $r = 0$ and $r = \infty$ is found to be

$$(4.7) \quad F = f_n(\rho)\, e^{in\theta}$$

where

$$(4.8) \quad f_n(\rho) = \int_0^{\rho} I_n(\rho')\, K_n(\rho)\, d\rho' + \int_{\rho}^{\infty} K_n(\rho')\, I_n(\rho)\, d\rho',$$

$$\rho = r/\varepsilon$$

Here, $I_n(z)$ and $K_n(z)$ are modified Bessel's functions. By superposition, we obtain

$$(4.9) \quad \Theta = i B \varepsilon^{-1} f_1(\rho)(e^{i\theta} - e^{-i\theta}),$$

$$(4.10) \quad \Phi = i B \varepsilon^{-1}[f_1(\rho)\, e^{-i\theta} + f_3(\rho)\, e^{-3i\theta}]$$

The integration can be carried out for $n=1$ so that

$$(4.11) \quad f_1(\rho) = \rho^{-1} - K_1(\rho)$$

$$(4.12) \quad f_3(\rho) = \int_0^{\rho} I_3(\rho')\, K_3(\rho)\, d\rho' + \int_{\rho}^{\infty} K_3(\rho')\, I_3(\rho)\, d\rho'$$

The stress field follows from (4.5)

$$(4.13) \quad \begin{aligned} t_{11} &= -\frac{\mu b}{4\pi(1-\nu)\varepsilon}\left[3 f_1(\rho) \sin\theta + f_3(\rho) \sin 3\theta\right], \\ t_{22} &= -\frac{\mu b\varepsilon}{4\pi(1-\nu)\varepsilon}\left[f_1(\rho) \sin\theta - f_3(\rho) \sin 3\theta\right], \\ t_{12} &= \frac{\mu b\varepsilon}{4\pi(1-\nu)\varepsilon}\left[f_1(\rho) \cos\theta + f_3(\rho) \cos 3\theta\right] \end{aligned}$$

In polar coordinates, components of the stress tensor follows from

$$(4.14) \quad t_{rr} + t_{\theta\theta} = \Theta, \qquad t_{\theta\theta} - t_{rr} + 2it_{r\theta} = \Phi\, e^{2i\theta}$$

Hence,

$$(4.15) \quad \begin{aligned} t_{rr} &= -\frac{\mu b}{4\pi(1-\nu)\varepsilon}\left[f_1(\rho) + f_3(\rho)\right] \sin\theta, \\ t_{\theta\theta} &= -\frac{\mu b}{4\pi(1-\nu)\varepsilon}\left[3f_1(\rho) - f_3(\rho)\right] \sin\theta, \\ t_{r\theta} &= \frac{\mu b}{4\pi(1-\nu)\varepsilon}\left[f_1(\rho) + f_3(\rho)\right] \cos\theta \end{aligned}$$

Again, we notice that the stress field vanishes at $r = 0$ so that contrary to the classical result, no

stress singularity is present at the center of the dislocation.

It should also be noted that (4.15) do not reduce to the formulas of the classical theory by setting $\varepsilon = 0$. This situation is, of course, well-known for singularly perturbed differential equations.

5. CONTINUOUS DISTRIBUTION OF DISLOCATIONS

A small neighborhood $n(\underset{\sim}{x})$ of $\underset{\sim}{x}$ in a *distorted body* V, may be relaxed to a small neighborhood $N(\underset{\sim}{X})$ of the image X of x, in an *undistorted* (or *natural*) configuration V, by releasing constraints exerted to $n(\underset{\sim}{x})$ by the rest of the body. A line element dx at $x \in n(x)$ can be expressed in terms of its image $d\underset{\sim}{X} \in N(\underset{\sim}{X})$ by

$$(5.1) \qquad d\underset{\sim}{x} = \underset{\sim}{A}\, d\underset{\sim}{X}$$

where $\underset{\sim}{A}(X)$ is called the *elastic distortion*. We assume that $A(X)$ is continuously differentiable and possesses a unique inverse, so that

$$(5.2) \qquad d\underset{\sim}{X} = \underset{\sim}{A}^{-1} d\underset{\sim}{x}$$

Consider a smooth surface S in V bounded by a closed curve C. The true Burger's vector $\underset{\sim}{b}$ of the dislocations piercing through S is defined by

$$(5.3) \qquad \underset{\sim}{b} = \oint_C d\underset{\sim}{X} = \oint_C \underset{\sim}{A}^{-1} d\underset{\sim}{x} = \int_S \underset{\sim}{a}\underset{\sim}{n}\, da$$

where $\underset{\sim}{n}$ is the unit normal to S, the positive sense of C being counter-clockwise, when sighting along n. Here, $\underset{\sim}{a}$ is called the *true dislocation density*

$$(5.4) \qquad \underset{\sim}{a} = \text{curl}\, \underset{\sim}{A}^{-1}, \qquad a_{jk} = \varepsilon_{kmn} A^{-1}_{jn,m}$$

For small distortions, we can write

$$(5.5) \qquad A_{kl} = \delta_{kl} + \alpha_{kl}, \qquad A^{-1}_{kl} \approx \delta_{kl} - \alpha_{kl}$$

so that

$$(5.6) \qquad a_{jk} = \varepsilon_{kmn} \alpha_{jm,n}$$

From this, it follows that

$$(5.7) \qquad a_{jk,k} = 0$$

The linear strain tensor e_{kl} and rotation tensor w_{kl} are given by

$$(5.8) \qquad e_{kl} = \frac{1}{2}(\alpha_{kl} + \alpha_{lk})$$

$$(5.9) \qquad w_{kl} = \frac{1}{2}(\alpha_{kl} - \alpha_{lk})$$

The strain incompatibility is expressed by

$$(5.10) \qquad \varepsilon_{ijk}\varepsilon_{lmn} e_{in,jm} = \eta_{kl}$$

where η_{kl} is called the *incompatibility tensor* and it is given by

$$(5.11) \qquad \eta_{kl} = \frac{1}{2}(\varepsilon_{kmn} a_{nl,m} + \varepsilon_{lmn} a_{nk,m})$$

All of these results are well-known in classical theory (cf. [16]).

Nonlocal theory stipulates that the stress strain relations is given by (2.1). For homogeneous, isotropic solids, under a mild assumption on the nature of the attenuating kernel, we have (2.12), i.e.

$$(5.12) \qquad (1 - \varepsilon^2\nabla^2)t_{kl} = \lambda e_{rr}\delta_{kl} + 2\mu e_{kl}$$

From this we solve for e_{kl}:

$$(5.13) \qquad e_{kl} = \frac{1}{2\mu}(1-\varepsilon^2\nabla^2)\left(t_{kl} - \frac{\nu}{1+\nu} t_{rr}\delta_{kl}\right)$$

where $\nu = \lambda/2(\lambda+\mu)$ is Poisson's ratio. If we substitute Eq. (5.13) into (5.10), we obtain

$$(5.14) \qquad (1-\varepsilon^2\nabla^2)\left[\nabla^2 t_{kl} + \frac{1}{1+\nu}(t_{rr,kl} - \nabla^2 t_{rr}\delta_{kl})\right] = 2\mu\,\eta_{kl}$$

These equations must be solved for t_{kl} which are subject to the equilibrium condition

$$(5.15) \qquad t_{kl,k} = 0$$

Following Kröner's classical approach [illegible], modifying the Beltrami solution of (5.15), we take

$$(5.16) \qquad t_{kl}/2\mu = \nabla^2\chi_{kl} + \frac{1}{1-\nu}(\chi_{rr,kl} - \nabla^2\chi_{rr}\delta_{kl})$$

where the symmetric stress function χ_{kl} is subject to

$$(5.17) \qquad \chi_{kl,l} = 0$$

Substituting (5.16) into (5.14), we obtain

$$(5.18) \qquad (1 - \varepsilon^2\nabla^2)\nabla^4\chi_{kl} = \eta_{kl}$$

Thus, given the dislocation density function a, through (5.11), we calculate the incompatibility tensor η_{kl}. The solution of (5.18) will then give χ_{kl}.

As expected, Eq. (5.18) reduces to the classical equation, when $\varepsilon = 0$. It is a singularly perturbed partial differential equation. To obtain the solution of (5.18), we must find Green's

function, $G_{k\ell mn}(x,\xi)$, which must satisfy

$$(5.19)\qquad (1-\epsilon^2\nabla^2)\nabla^4 G_{k\ell mn} = \delta(x-\xi)\,\delta_{k\ell}\,\delta_{mn}$$

When $G_{k\ell mn}$ is known, then the solution of (5.18) is given by

$$(5.20)\qquad \chi_{k\ell} = \int_V G_{k\ell mn}(x,\xi)\,\eta_{mn}(\xi)\,dv(\xi)$$

In the case of the *plane strain*, introducing Airy's stress function $\Phi(x_1,x_2)$ by

$$(5.21)\qquad t_{11}=\Phi_{,22}, \quad t_{22}=\Phi_{,11}, \quad t_{12}=-\Phi_{,12}$$

we obtain an equation replacing (5.18)

$$(5.22)\qquad (1-\epsilon^2\nabla^2)\nabla^4\Phi = 2\mu\eta$$

where

$$(5.23)\qquad \eta=\eta_{33}=a_{23,1}-a_{13,2}$$

$$(5.24)\qquad a_{23}=\alpha_{21,2}-\alpha_{22,1}, \quad a_{13}=\alpha_{11,2}-\alpha_{12,1}$$

depend on x_1 and x_2 only.

In the case of the *anti-plane strain*, equations of equilibrium are satisfied if

$$(5.25)\qquad t_{13}=\frac{\partial\phi}{\partial x_2}, \qquad t_{23}=-\frac{\partial\phi}{\partial x_1}$$

and we obtain

$$(5.26)\qquad (1-\epsilon^2\nabla^2)\nabla^2\phi = \mu a_{33}$$

where

$$(5.27)\qquad a_{33}=\alpha_{31,2}-\alpha_{32,1}.$$

6. GREEN'S FUNCTIONS: STATE OF STRESS

The determination of the stress fields arising from the continuous distributions of dislocations requires that we obtain the Green function for the stress functions $\chi_{k\ell}$, Φ and ϕ. Here we determine Green's functions for solids of infinite extent.

(i) THREE-DIMENSIONAL INFINITE SOLID - The operator ∇^2 is invariant under the rotations of coordinates. For the infinite space, we seek a solution of (5.16) which depends upon $|x-\xi|$ only, i.e.

$$(6.1)\qquad (1-\epsilon^2\nabla^2)\nabla^4 G = \delta(x-\xi)$$

Since the operators $(1-\epsilon^2\nabla^2)$ and ∇^4 commutes, we set

$$(6.2)\qquad (1-\epsilon^2\nabla^2)G = H, \qquad \nabla^4 H = \delta(x-\xi)$$

For the infinite space, H is given by

$$(6.3)\qquad H = -|x-\xi|/8\pi$$

In spherical coordinates, using the operator

$$\nabla^2 = r^{-2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)$$

we obtain

$$(6.4)\qquad G(|x-\xi|) = \frac{\epsilon^2}{4\pi|x-\xi|}\exp(-|x-\xi|/\epsilon) - \frac{|x-\xi|}{8\pi}, \qquad \epsilon\neq 0$$

$$(6.5)\qquad G(|x-\xi|) = -|x-\xi|/8\pi, \qquad \epsilon=0$$

where we also determined a constant to render t_{kk} regular at $x=\xi$.

The solution of (5.16) for the infinite media, is given by

$$(6.6)\qquad \chi_{k\ell} = \int_V G(|x-\xi|)\,\eta_{k\ell}(\xi)\,dv(\xi)$$

which satisfies conditions (5.17) on account of (5.7) and (5.11). Upon substituting from (5.11), this may be expressed as

$$(6.7)\qquad \chi_{k\ell}(x) = \frac12\epsilon_{ijk}\int_V a_{j\ell}(\xi)\frac{\partial G}{\partial x_i}\,dv(\xi) + \frac12\epsilon_{ij\ell}\int_V a_{jk}(\xi)\frac{\partial G}{\partial x_i}\,dv(\xi)$$

where we used the Green-Gauss theorem and set a surface term at infinity to zero.

Eq. (6.7) may be used to study various special cases involving surface and line distributions of dislocations. For example, for a line distribution of dislocation along a closed curve C, we obtain

$$(6.8)\qquad \chi_{k\ell} = \frac12\epsilon_{ijk}b_j\oint_C\frac{\partial G}{\partial x_i}\,ds_\ell + \frac12\epsilon_{ij\ell}b_j\oint_C\frac{\partial G}{\partial x_i}\,ds_k$$

where b_j is the Burger's vector per unit length of C and ds_j is the element of arc.

The stress field due to a line distribution of dislocation is obtained by substituting (6.5) into (5.16)

$$(6.9)\quad t_{k\ell}/2\mu = \frac{1}{2}\epsilon_{rij}b_j \oint_C [\nabla^2 G_{,i}(\delta_{r\ell}ds_k + \delta_{rk}ds_\ell)$$
$$+ \frac{2}{1-\nu}(G_{,k\ell i} - \nabla^2 G_{,i}\delta_{k\ell})ds_r]$$

This result is identical to the Peach-Koehler[15,16] formula with the modification that here G is the nonlocal Green's function (6.4) with $\epsilon \neq 0$. The most interesting new feature of (6.9) is that it does not exhibit unbounded stress fields and energies at any point on C or elsewhere.

(ii) TWO-DIMENSIONAL INFINITE PLANE - Green's function in this case can be found to be similar to decomposition (6.2) with ∇^2 and H, given by

$$(6.10)\qquad \nabla^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right),$$

$$(6.11)\qquad H = -\frac{1}{2\pi}\ln|\underset{\sim}{x}-\underset{\sim}{\xi}|$$

Hence,

$$(6.12)\quad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = \frac{1}{2\pi}K_0(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\epsilon) - \frac{(\underset{\sim}{x}-\underset{\sim}{\xi})\cdot(\underset{\sim}{x}-\underset{\sim}{\xi})}{8\pi\epsilon^2}\ln(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\epsilon),\quad \epsilon \neq 0$$

$$(6.13)\quad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{(\underset{\sim}{x}-\underset{\sim}{\xi})\cdot(\underset{\sim}{x}-\underset{\sim}{\xi})}{8\pi}\ln(|\underset{\sim}{x}-\underset{\sim}{\xi}|),\quad \epsilon = 0$$

where $K_0(z)$ is the modified Bessel's function.

Airy's stress fucntion is obtained to be

$$(6.14)\quad \phi(\underset{\sim}{x}) = 2\mu\int_S\left[\frac{\partial G}{\partial x_1}a_{23}(\underset{\sim}{\xi}) - \frac{\partial G}{\partial x_2}a_{13}(\underset{\sim}{\xi})\right]da$$

where we used the Green-Gauss theorem and set a line integral to zero at infinity. For a *line distribution* of dislocations in the $x_3 = 0$-plane, we will have

$$(6.15)\quad \phi(\underset{\sim}{x}) = -2\mu\oint_C\left[\frac{\partial G}{\partial x_1}b_2(\underset{\sim}{\xi})\,d\xi_1 + \frac{\partial G}{\partial x_2}b_1(\underset{\sim}{\xi})\,d\xi_2\right]$$

The stress is calculated by using (5.21)

$$t_{11} = -2\mu\oint_C(G_{,122}b_2d\xi_1 + G_{,222}b_1\,d\xi_2),$$

$$(6.16)\quad t_{22} = -2\mu\oint_C(G_{,112}b_2d\xi_1 + G_{,211}b_1\,d\xi_2),$$

$$t_{12} = 2\mu\oint_C(G_{,112}b_2d\xi_1 + G_{,212}b_1\,d\xi_2)$$

where indices after a comma denote partial derivatives with respect to x_k, e.g.

$$G_{,122} = \partial^3 G/\partial x_1\partial x_2^2$$

(iii) ANTI-PLANE STRAIN - The Green function for the differential operator in (5.24) is obtained to be

$$(6.17)\quad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{1}{2\pi}[\ln(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\epsilon) + K_0(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\epsilon)],\qquad \epsilon \neq 0$$

$$(6.18)\quad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{1}{2\pi}\ln|\underset{\sim}{x}-\underset{\sim}{\xi}|,\qquad \epsilon = 0$$

The stress function ψ is given by

$$(6.19)\quad \psi(\underset{\sim}{x}) = \mu\int_S G(|\underset{\sim}{x}-\underset{\sim}{\xi}|)\,a_{33}(\underset{\sim}{\xi})\,da$$

The stress field is found to be

$$(6.20)\quad t_{13} = \mu\int_S\frac{\partial G}{\partial x_2}b(\underset{\sim}{\xi})\,d\xi_1 d\xi_2,$$
$$t_{23} = -\mu\int_S\frac{\partial G}{\partial x_1}b(\underset{\sim}{\xi})\,d\xi_1 d\xi_2$$

For a line distribution of dislocations on the $x_3 = 0$-plane, the stress field is given by

$$(6.21)\quad t_{13} = \mu\oint_C\frac{\partial G}{\partial x_2}b(\underset{\sim}{\xi})\,ds,$$
$$t_{23} = -\mu\oint_C\frac{\partial G}{\partial x_1}b(\underset{\sim}{\xi})\,ds$$

Eringen

In plane polar coordinates (r,θ), we have

$$(6.22)\qquad t_{zr} = \mu \int_C \frac{1}{r}\frac{\partial G}{\partial \theta}\, b(\xi)\, ds$$

$$t_{z\theta} = -\mu \int_C \frac{\partial G}{\partial r}\, b(\xi)\, ds$$

7. STRESS DISTRIBUTIONS

Here, I present some results on the stress field due to continuous distributions of dislocations along a line segment.

(i) EDGE DISLOCATION ALONG A LINE SEGMENT - The stress field due to an edge dislocation can be calculated by using (6.16). In this case, $b_2 = 0$ and we consider $b_1 =$ const. distributed along a line segment $x_2 = 0$, $|x_1| < \ell$. Using the fact that $G_{,2} = \partial G/\partial x_2 = -\partial G/\partial \xi_2$, the integrations in (6.16) are performed readily

$$(7.1)\qquad t_{11} = 2\mu b_1 [G_{,22}(\rho_1) - G_{,22}(\rho_2)],$$

$$t_{22} = 2\mu b_1 [G_{,11}(\rho_1) - G_{,22}(\rho_2)],$$

$$t_{12} = -2\mu b_1 [G_{,12}(\rho_1) - G_{,12}(\rho_2)]$$

where

$$(7.2)\qquad \rho_1 = [(x_1-\ell)^2 + x_2^2]^{1/2}/\epsilon ,$$

$$\rho_2 = [(x_1+\ell)^2 + x_2^2]^{1/2}/\epsilon$$

Green's function G is given by (6.12).

(ii) SCREW DISLOCATION - For a uniform distribution of screw dislocations along a line segment $x_2 = 0$, $|x_1| < \ell$, through (6.21), we find that

$$(7.3)\qquad t_{23} = \mu b[G(\rho_1) - G(\rho_2)]$$

where G is given by (6.17) and ρ_1 and ρ_2 by (7.2).

If the number of dislocations is N over a distance ℓ and b_0 is the atomic Burger's vector, we have for the macroscopic Burger's vector,

$$(7.4)\qquad b = b_0 N/2\ell$$

The shear stress given by (7.3) may be expressed in non-dimensional form

$$(7.5)\qquad T_2 = t_{23}/t_d = \ln[(x+1)/(x-1)] + K_0(Y|x+1|) - K_0(Y|x-1|)$$

where

$$(7.6)\qquad t_d = \mu b/2\pi = \mu b_0 N/2\pi\ell ,$$

$$x = x_1/\ell , \quad Y = \ell/\epsilon$$

(iii) SINGLE SCREW - For a single screw, the shear stresses can be obtained by substituting

$$(7.7)\qquad b(\xi) = b_0\, \delta(\xi)$$

into (6.20), where $\delta(\xi)$ is the Dirac-delta measure. This leads to

$$(7.8)\qquad t_{13} = \mu b_0 \frac{\partial G}{\partial x_2} = -\frac{\mu b_0}{2\pi}\frac{x_2}{r^2}\left[1 - \frac{r}{\epsilon}K_1(r/\epsilon)\right],$$

$$t_{23} = -\mu b_0 \frac{\partial G}{\partial x_1} = \frac{\mu b_0}{2\pi}\frac{x_1}{r^2}\left[1 - \frac{r}{\epsilon}K_1(r/\epsilon)\right]$$

where $r = (x_1^2 + x_2^2)^{1/2}$. This result is identical to Eq. (5.5) obtained differently, in polar coordinates.

For a single screw, the shear stress Eq. (5.5) may be expressed in non-dimensional form

$$(7.9)\qquad T_\theta(\rho) = (2\pi\epsilon/\mu b)\, t_{z\theta} = \rho^{-1}[1 - \rho K_1(\rho)]$$

where

$$(7.10)\qquad \rho = r/\epsilon .$$

The stress field given by Eq. (7.9) is displayed graphically in Fig. 1. It has no singularity. In fact, contrary to the prediction made in classical elasticity, $T_\theta(\rho)$ vanishes at $\rho = 0$. The maximum stress occurs at $\rho \approx 1.1$ and is given by

$$(7.11)\qquad t_{z\theta\, max} = 0.3993\, \frac{b}{2\pi\epsilon}$$

The fracture will begin when $t_{z\theta\, max} = t_c$ = the cohesive stress that holds atomic bonds together. Therefore, nonlocal theory predicts that

(a) *Fracture begins at a short distance $\rho \approx 1.1$ from the center of dislocation.*

Note that while this is only atomic distances away for single crystals, nevertheless, it may be a

finite distance for amorphous and composite materials. It depends on the internal characteristic length a (more precisely, $e_0 a$).

(b) *Fracture Criterion: The fracture occurs when the maximum shear stress (given by (7.11)) reaches the value of the cohesive stress that holds atomic bonds together.*

With this criterion, the maximum stress hypothesis for fracture is restored for microscopic and atomic phenomena as well.

If we write $h = \varepsilon/0.3993$, Eq. (7.11) agrees with Frenkel's estimate of the theoretical strength of single crystals, based on atomic considerations (cf. Kelly [18], p. 12). In fact, if we use $\varepsilon = e_0 a = 0.39a$, we find for the single aluminum crystal,

$$t_c/\mu = 0.12 \qquad \{Al: [111] < 1\bar{1}0 >\} \tag{7.12}$$

This is very close to the theoretical strength $t_y/\mu = 0.11$ based on atomic models.

In the case of line distribution of screws with constant Burgers vector, the non-dimensional shear stress is displayed in Fig. 2 for various values of γ. Here we observe that the shear stress is maximum near the end points of the line segment $x_1 = \pm \ell$, $x_2 = 0$. It is located slightly outside of the end points and for $\gamma \geq 3$ it is very close to the end points (see Table 1). Again, contrary to the classical elasticity solution, there is no singularity at $x_1 = \pm \ell$, $x_2 = 0$. Behavior of T_2 is governed basically by the first-term in (7.5) except near $x = \pm 1$. At $x = 1$, we have

$$t_{23} = \frac{\mu b_0 N}{2\pi\varepsilon}\,\frac{\ln\gamma}{\gamma} = \frac{\mu B}{2\pi h} \tag{7.13}$$

If we write

$$B = b_0 N, \qquad h = \varepsilon\gamma/\ln\gamma \tag{7.14}$$

Eq. (7.13) may be interpreted in terms of the slip of atomic layers of distance h by one *macroscopic* dislocation of Burger's vector B. The ratio of the cohesive stress t_d for the line distribution to that of a single screw is given by

Table 1: Maximum Shear Stress and its Location for a Uniform Distribution of Screw Dislocations Along a Straight Line Segment

γ =	1	1.5	2	3	5	10
x =	1.446	1.197	1.103	1.039	1.009	1.000
T_{2max} =	0.7478	1.0501	1.3008	1.6851	2.3026	2.9957

$$\frac{t_d}{t_c} = \frac{N}{0.3993}\,\frac{[illegible]}{[illegible]} \tag{7.15}$$

This gives the shear stress reduction due to the presence of $2N$ dislocations distributed uniformly along a straight line segment of length 2ℓ. Since $t_d \leq t_c$, the maximum number of dislocations is given by

$$N_{max} = 0.3993\,\frac{[illegible]}{\ln[illegible]} \tag{7.16}$$

Of course, this number will have to be modified when the distribution is not uniform.

The stress fields due to a uniform distribution of screw dislocations along a circle are given in another publication.

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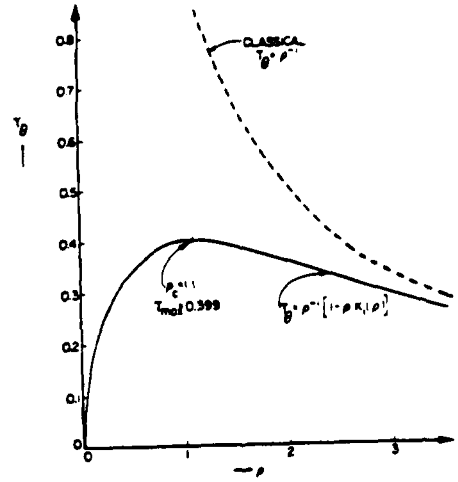

Fig. 1: NON-DIMENSIONAL HOOP STRESS FOR SCREW DISLOCATION

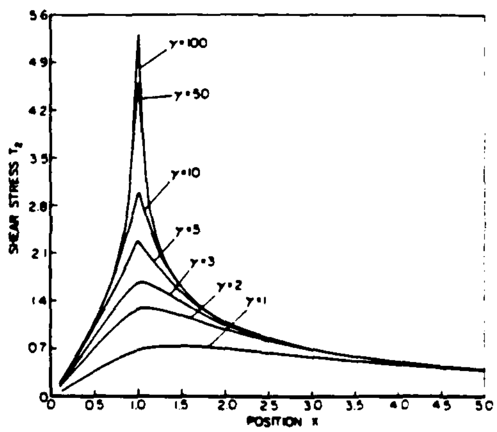

Fig. 2: SHEAR STRESS DISTRIBUTION (Line Segment)

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